Differentiation Techniques

Chain/Product/Quotient Rule

2004	**D2.** For the function defined by $y = x^2 \ln x,\ x > 0$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Hence show that $x\frac{d^2y}{dx^2} - \frac{dy}{dx} = kx$, stating the value of the constant k.	**4** **2**
2005 SP	**11.** Differentiate with respect to x (*a*) $f(x) = (1 + 2x)\ln(1 + 2x),\ x > -\frac{1}{2}$, (*b*) $g(x) = e^{\cot 2x},\ 0 < x < \frac{\pi}{2}$.	 **3** **2**
2005	**B1.** Differentiate, and simplify as appropriate, (*a*) $f(x) = \exp(\tan \frac{1}{2}x)$, where $-\pi < x < \pi$, (*b*) $g(x) = (x^3 + 1)\ln(x^3 + 1)$, where $x > 0$.	 **3** **3**
2006	**B2.** Given that $y = \ln(1 + \sin x)$, where $0 < x < \frac{\pi}{2}$, show that $\frac{d^2y}{dx^2} = \frac{-1}{1+\sin x}$.	**5**
2008	**B2.** Given that $y = e^{2x}\cos x$, find $\frac{dy}{dx}$.	**3**
2010	**B1.** Differentiate the following, simplifying your answers as appropriate. (*a*) $f(x) = e^{2x}\tan x,\ \frac{-\pi}{2} < x < \frac{\pi}{2}$. (*b*) $g(x) = \frac{\cos 2x}{x^3}$.	 **3** **4**
2011	**B1.** Differentiate the following functions, simplifying where possible: (*a*) $f(x) = \frac{1+\sin x}{1+2\sin x},\ 0 \le x \le \pi$; (*b*) $g(x) = \ln(1 + e^{2x})$.	 **3** **2**
2012	**B2.** (*a*) Given the curve $y = \frac{x}{x^2 + 4}$, calculate the gradient when $x = 2$. (*b*) Determine $\int e^{-2t}\,dt$.	**3** **2**
2013	**B1.** Given that $y = \sin(e^{5x})$, find $\frac{dy}{dx}$.	**2**
2014	**B1.** Find the gradient of the tangent to the curve $y = 2x\sqrt{x-1}$ at the point where $x = 10$.	 **4**
2015	**B1.** Given that $y = e^{5x}\tan 2x$, find $\frac{dy}{dx}$.	**3**

2015	**B3.** A curve is defined by $$y = \frac{\sin x}{2 - \cos x} \text{ for } 0 \le x \le \pi.$$ Find the exact values of the coordinates of the stationary point of this curve.	**5**
2016 EX	**2.** Differentiate the function $g(x) = \dfrac{\cos 2x}{x^3}$, simplifying your answer as appropriate.	**4**
2016 SP	**2.** Given $y = e^{x^2} \cos x$ find $\dfrac{dy}{dx}$.	**3**
2016	**4.** Find the equation of the tangent to the curve $y = x \ln x$ at the point where $x = e$.	**3**
2017	**2.** (a) If $f(x) = \dfrac{\ln x}{2x^2}$, $x \neq 0$, find $f'(x)$. Fully simplify your answer. (b) If $y = \text{cosec}^2 3x$, show that $$\frac{dy}{dx} + 6y \cot 3x = 0.$$	**3** **3**

Parametric Differentiation

2005	**B3.** A curve is defined by the parametric equations $x = 5t^2 - 5$, $y = 3t^3$. Find the value of t corresponding to the point $(0, -3)$ and calculate the gradient of the curve at this point.	**2, 3**
2007	**B3.** A curve is defined parametrically by $x = \dfrac{t}{t^2 + 1}$, $y = \dfrac{t-1}{t^2+1}$. Obtain $\dfrac{dy}{dx}$ as a function of t.	**5**
2009	**B3.** A particle moves along a curve in the x-y plane. The curve is defined by the parametric equations $$x = t^2 + 1, \qquad y = 1 - 3t^3,$$ where t is the time elapsed since the start. Find $\dfrac{dy}{dx}$ in terms of t and hence obtain an equation of the tangent to the curve when $t = 2$.	**5**
2011	**B3.** A curve is defined by the equations $$x = 5\cos t \qquad \text{and} \qquad y = 3\sin t, \qquad (0 \le t < 2\pi).$$ Find the gradient of the curve when $t = \dfrac{\pi}{6}$.	**4**

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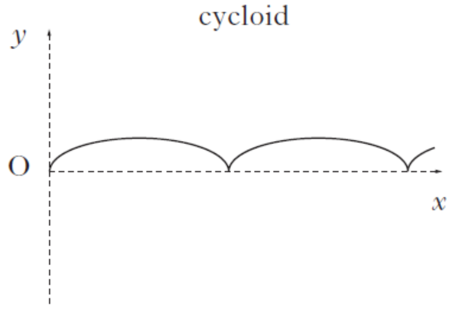
<tr><td>2013</td><td>

B6. The cycloid curve below is defined by the parametric equations

$$x = t - \sin t,\ y = 1 - \cos t.$$

cycloid



(*a*) Find $\dfrac{dy}{dx}$ in terms of t. **2**

(*b*) Show that the value of $\dfrac{d^2y}{dx^2}$ is always negative, in the case where $0 < t < 2\pi$. **5**

(*c*) A particle follows the path of the cycloid where t is the time elapsed since the particle's motion commenced.
Calculate the speed of the particle when $t = \dfrac{\pi}{3}$. **2**

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<tr><td>2016 EX</td><td>

6. A particle moves along a curve in the x–y plane. The curve is defined by the parametric equations $x = t^2 + 1$, $y = 1 - 3t^3$, where t is the time elapsed since the start of the motion.

Find $\dfrac{dy}{dx}$ in terms of t and hence obtain the equation of the tangent to the curve when $t = 2$. **4**

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<tr><td>2016</td><td>

10. A stone is thrown from the top of a cliff and the subsequent motion can be modelled in the $x\,y$ plane by the equations $x = 4t$ **and** $y = 20 + 2t - 5t^2$.

(a) Use parametric differentiation to find $\dfrac{dy}{dx}$ in terms of t. **2**

(b) (i) Find the angle of projection of the stone. **2**

(ii) By considering $\dfrac{dy}{dx}$ find the value of t when the stone is moving at 45° below the horizontal. **2**

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Implicit Differentiation

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<tr><td>2016 SP</td><td>

7. Calculate the gradient of the tangent to the curve $xy^2 - 4xy = 5$ at the point (1,5). **4**

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<tr><td>2017</td><td>

11. A curve is defined by $3y^2 - x^2y = 4,\quad x \geq 0,\ y \geq \dfrac{2}{\sqrt{3}}$.

Use implicit differentiation to find the gradient of the tangent when $x = 2$. **5**

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